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Preparation of Schrödinger cat states with cold ions beyond the Lamb-Dicke limit

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Abstract

A scheme for preparing Schrödinger cat (SC) states is proposed beyond the Lamb-Dicke limit in a Raman-Λ-type configuration. It is shown that SC states can be obtained more efficiently with our scheme than with the former ones.

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In recent years the system of cold ions moving in a harmonic trap is considered to be a prospective physical setting for the preparation and investigation of nonclassical states [1−3]. It has been proven that, in the limit where coherent interaction can dominate over dissipative process, the model of a cold ion strongly coupled to a harmonic oscillator is formally similar to the cavity quantum electrodynamics (QED).

Almost all schemes for the preparation of nonclassical motional states of a trapped ion are based on the Lamb-Dicke limit(LDL) under the weak excitation regime(WER), which correspond to the actual case in the present ion-trap experiments [4−6]. The LDL means that the spatial dimensions of the ground motional state are much smaller than the effective wavelength of the laser wave, and the WER is the condition that the Rabi frequency describing the laser-ion interaction is much smaller than the trap frequency. In the case of the LDL and WER, Jaynes-Cummings model(JCM)[7,8] can be used to describe the trapped ion system in the supposition that the ion is of two levels, the trap's potential can be quantized to be a harmonic oscillator, and the radiating lasers can be taken as the classical forms of standing or traveling waves. Some techniques developed in the framework of cavity QED based on JCM can be immediately transcribed to the ion trap system by taking advantage of the analogy between the cavity QED and the ion trap problem. Recently, a scheme [2,9] for the preparation of Schrödinger cat (SC) states was proposed under the strong excitation regime (SER), opposite to the WER. As in the SER the Rabi frequency is very large, the operation time for the state preparation can be much reduced, which is advantageous to avoid the decoherence. On the other hand, we also noted that the case beyond the LDL has been discussed intensively for the rapid laser-cooling of the ion [10−12]. It has been shown that the laser-ion interaction beyond the LDL in the manipulation of the cold ions is helpful for reducing the noise in the trap, loosing the confinement of the trap and improving the cooling rate[12]. However, as far as we know, no specific proposal has been put forward so far for preparing nonclassical states of the ion beyond the LDL.

In this letter, we try to prepare the SC states [13] under the WER, but beyond the LDL. The SC state, i.e., the superposition of macroscopically distinguishable states, has

been drawn much attention over past several decades due to both the SC paradox, i.e., a Schrödinger's thought experiment, and the possibility of experimental realization in the mesoscopic system. Under both the LDL and WER, there have been some proposals [14−16] for the preparation of SC states with trapped cold ions. Experimentally, SC states have been obtained in NIST group with single cold $^{9}Be^{+}$[5]. We will show that, beyond the LDL, the preparation of SC states can be made more rapidly and simply than those within the LDL. From the viewpoint of decoherence, it may be of importance for the experimental implementation and measurement due to the reduction of the operation time.

We investigate the situation that the single ultracold ion radiated by lasers in the Raman-Λ-type configuration[4]. The electronic structure is employed with two lower levels $|e>$ and $|g>$ coupled to a common upper state $|r>$, and the two lasers with frequencies ω_1 and ω_2 respectively are assumed to propagate along opposite direction. For a sufficiently large detuning to the level $|r>$, $|r>$ may be adiabatically eliminated, and what we have to treat is an effective two-level system, in which the lasers drive the electric-dipole forbidden transition $|g>\leftrightarrow|e>$. The dimensionless Hamiltonian of such a system in the frame rotating with the effective laser frequency $\omega_l(=\omega_1-\omega_2)$ can be written as[9]

$$H=\frac{\Delta}{2}\sigma_z+a^+a+\frac{\Omega}{2}[\sigma_+e^{i\eta(a^++a)}+\sigma_-e^{-i\eta(a^++a)}] \tag{1}$$

where the detuning $\Delta=(\omega_0-\omega_l)/\nu$ with ω_0 being the transition frequency of two levels of the ion, and ν the frequency of the trap. Ω is the Rabi frequency and η the effective Lamb-Dicke parameter given by $\eta=\eta_1+\eta_2$ with subscripts denoting the counterpropagating laser field. σ_i $(i=\pm,z)$ are Pauli operators, and a^+ and a are operators of creation and annihilation of the phonon field, respectively. The notations '+' and '-' in front of $i\eta(a^++a)$ correspond to the absorption of a photon from one beam followed by emission into the other beam and vice versa, respectively. ν is generally supposed to be much greater than the atomic decay rate, called the strong confinement limit, for neglecting the effect of the atomic decay. We first perform following unitary transformations on Eq.(1), that is[17]

$$H^I=THT^+=\frac{\Omega}{2}\sigma_z+a^+a-i\xi(a^+-a)\sigma_x-\epsilon\sigma_x+\xi^2 \tag{2}$$

where $T = \frac{1}{\sqrt{2}} \begin{pmatrix} D^+ & D \\ -D^+ & D \end{pmatrix}$ with $D = e^{i\xi(a^+ + a)}$, $\xi = \eta/2$, $\epsilon = \Delta/2$, and $\sigma_x = \sigma_+ + \sigma_-$. As we suppose $\Omega \ll 1 \leq \xi$, Eq.(2) can be reduced to

$$H^I = a^+a - i\xi(a^+ - a)\sigma_x - \epsilon\sigma_x + \xi^2 \tag{3}$$

where the detuning term ϵ is retained due to no special requirement on it. Therefore, the time evolution operator in the original representation is

$$\hat{U}(t) = T^+ \exp(-iH^I t)T$$

$$= \frac{1}{2} e^{-i\xi^2 t} \begin{pmatrix} D & -D \\ D^+ & D^+ \end{pmatrix} e^{-it[a^+a - i\xi(a^+ - a)\sigma_x - \epsilon\sigma_x]} \begin{pmatrix} D^+ & D \\ -D^+ & D \end{pmatrix}. \tag{4}$$

Direct algebra on Eq.(4) yields

$$\hat{U}(t) = \frac{1}{2} e^{-i\xi^2 t} \begin{pmatrix} e^{-ia^+at} D e^{-\xi t(a^+ - a)} & -e^{-ia^+at} D e^{-\xi t(a^+ - a)} \\ e^{-ia^+at} D^+ e^{\xi t(a^+ - a)} & e^{-ia^+at} D^+ e^{\xi t(a^+ - a)} \end{pmatrix} \times$$

$$\begin{pmatrix} cosh[\xi(a^+ - a)t] & -sinh[\xi(a^+ - a)t] \\ -sinh[\xi(a^+ - a)t] & cosh[\xi(a^+ - a)t] \end{pmatrix} \times$$

$$\begin{pmatrix} \cos[\frac{\xi}{2}t^2(a^+ + a)] & -i\sin[\frac{\xi}{2}t^2(a^+ + a)] \\ -i\sin[\frac{\xi}{2}t^2(a^+ + a)] & \cos[\frac{\xi}{2}t^2(a^+ + a)] \end{pmatrix} \begin{pmatrix} \cos\epsilon t & i\sin\epsilon t \\ i\sin\epsilon t & \cos\epsilon t \end{pmatrix} \begin{pmatrix} D^+ & D \\ -D^+ & D \end{pmatrix} \tag{5}$$

in which we have used Baker-Campbell-Hausdorff theorem, and formulas $e^{iA\sigma_x} = \cos A + i\sigma_x \sin A$ and $e^{A\sigma_x} = coshA + \sigma_x sinhA$.

Consider that the ion has been laser-cooled to the dark state $|g>|0>$ beyond the LDL[10−12], where we denote the electronic and motional ground states by $|g>$ and $|0>$ respectively. Defining $|e> = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $|g> = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, a laser pulse $\hat{V} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ applied on the ion will yield the state $\Psi_1 = \frac{1}{\sqrt{2}}(|e> + |g>)|0>$. Then performing $\hat{U}(t)$ on Ψ_1, we have the superposition of coherent states correlated with the internal states of the ion

$$\Psi_2 = \frac{1}{\sqrt{2}} e^{-i\xi^2 t}[e^{-i\epsilon t}|e>|i\frac{\xi}{2}t^2 e^{-it}> + e^{i\epsilon t}|g>| - i\frac{\xi}{2}t^2 e^{-it}>]. \tag{6}$$

Finally, we apply $\hat{V}$ once again, which produces

$$\Psi_3 = \frac{1}{\sqrt{2}} e^{-i\xi^2 t}(\Phi_+|e> + \Phi_-|g>) \tag{7}$$

with the SC states $\Phi_\pm = \frac{1}{\sqrt{2}}(e^{i\epsilon t}| - i\frac{\xi}{2}t^2 e^{-it} > \pm e^{-i\epsilon t|}|i\frac{\xi}{2}t^2 e^{-it} >)$.

Let us take more specific consideration on Eq.(7). To measure the SC states perfectly, we can use the technique of electronic shelving amplification[18]. By introducing the fourth electronic level $|f>$ of the ion, and a weak laser beam resonant with the transition of $|g> \rightarrow |f>$, we can obtain Φ_+ perfectly corresponding to no fluorescence in observation. However, to obtain a perfect Φ_-, we have to modify the last step of above preparation process, that is, replacing $\hat{V}$ with $\hat{V}' = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. Then we have

$$\Psi_3' = \frac{1}{\sqrt{2}} e^{-i\xi^2 t}(-\Phi_-|e> + \Phi_+|g>). \tag{8}$$

So the perfect Φ_- can be obtained similarly in the absence of fluorescence. Another problem is that, $\Phi_\pm$ should be macroscopic in the sense that the component states $| - i\frac{\xi}{2}t^2 e^{-it} >$ and $|i\frac{\xi}{2}t^2 e^{-it} >$ are distinguishable. Unfortunately, if $t = k\pi$ with $k = 0, 1, 2, \cdots$, SC states $\Phi_\pm = \frac{1}{\sqrt{2}}[e^{i\epsilon k\pi}| - (-1)^k i\frac{\xi}{2}k^2\pi^2 > \pm e^{-i\epsilon k\pi}|(-1)^k i\frac{\xi}{2}k^2\pi^2 >]$ can not be observed directly since its probability distribution in the position representation has only one peak centred at $< R >= 0$[2]. So we had better choose the time $t = \frac{1}{2}(2k+1)\pi$ with $k = 0, 1, 2, \cdots$. The SC state at this moment is of the form

$$\Phi_\pm = \frac{1}{\sqrt{2}}[e^{i\frac{\xi}{2}(2k+1)\pi}| - (-1)^k \frac{1}{8}\xi(2k+1)^2\pi^2 > \pm e^{-i\frac{\xi}{2}(2k+1)\pi}|(-1)^k \frac{1}{8}\xi(2k+1)^2\pi^2 >] \tag{9}$$

which has two maxima in the position representation centred at $< R >\sim \pm\frac{1}{8}\xi(2k+1)^2\pi^2$. As distinguishing the two component coherent states needs the two peaks to be spatially separated by more than a wavelength, we should wait for at least $t \geq \sqrt{2\pi/\xi}$ when applying $\hat{U}(t)$ on the ion.

The form of Eq.(7) is very similar to the experimental results in Ref.[5]. However, as the laser-cooling of the ion beyond the LDL to the motional ground state is more rapid than the case under the LDL[12], and the procedure of preparing the SC state in our scheme is simpler than that in Ref.[5], our scheme is obviously more efficient. In fact, comparing

with former various schemes, our scheme is also more efficient in the preparation of the SC state. In Refs.[2,9], the laser pulses should be applied on the ion alternatively from opposite directions for many times in order to obtain a observable SC state. In contrast, we only have three steps of the laser-ion interaction, i.e., $\hat{V}\hat{U}(t)\hat{V}$ or $\hat{V}\hat{U}(t)\hat{V}'$, and a suitable choice of t. In Ref.[14], the SC state is obtained as the final result of the spontaneous emission process. So the state preparation will take a long time if the trapped ion is nearly isolated from the external environment. Moreover, as Refs.[15,16] describe the process under the LDL and WER, it is easily found that preparing observable SC states with those schemes is also time-consuming. On the contrary, in our scheme, as η is large, and the values of the component coherent states are proportional to t^2, the time for preparing a observable SC state is much shorter than that in above schemes.

More specifically, even if we neglect the speed-up in the laser-cooling of the trapped ion beyond the LDL, following simple numerical estimates for the preparation time of the observable SC states can also show the advantage of our scheme. With current experimental parameters of $\eta = 0.202$, (dimensionless) $\Omega = 0.1$ and $\nu = 10^7 Hz$[4,5], we obtain that the dimensionless time for preparing a observable SC state with the scheme in [2] is $\frac{\pi}{4}(\sqrt{\frac{\pi}{2\eta^2}}-1) = 4.09$ in the supposition that the delay time between any two operations of laser pulses can be omitted, and the time corresponding to [16] is $\frac{\pi e^{\eta^2/2}}{\eta\Omega} = 159$. If we choose the scheme in [14] and the cold ion $^9Be^+$, the time will be much longer since the metastable level of $^9Be^+$ is $1 \sim 10$ seconds. So the minimum dimentionless time is 10^7. In contrast, with our scheme, $t \sim \sqrt{4\pi/\eta} = 2.51$ if $\eta = 2.0$. In fact, η can be maximized to 3.0 [10,12], so t can be minimized to 2.05.

In summary, a simple but efficient scheme for preparing the SC states of motion of a cold trapped ion has been proposed based on the ion under the WER but beyond the LDL. As far as we know, it is the first proposal for the preparation of nonclassical motional states of the cold ions beyond the LDL. As the trapped ions beyond the LDL are less tightly confined and more rapidly laser-cooled, which meets the requirement of the ion trap quantum computing[19], the investigation on the laser-ion interaction beyond the LDL is

of importance. We also note a proposal using SC states to be the robust qubits for the quantum computing[20]. Therefore we believe the present work would be helpful for the future exploration of the ion-trap system beyond the LDL, although the experimental work in this respect has not been reported yet.

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